

# BigSEM for network data

We will show how to use BigSEM to analyze network data in the SEM framework.

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# SEM with networks - background

Network data can be integrated into the SEM framework in different ways. We focus on two main approaches here. The first approach extracts the information from a network based on each participant and then use that information as variable(s) in a SEM model. In this method, each participant (node) in the network is the basic unit for analysis. The second approach extracts information from a network based on each relationship present. In this method, each pair of participants or nodes are used as the basic unit for analysis.

In our software, we propose and implement four types of models.

## Network nodes as analysis units

In this method, each participant is treated as the basic unit of analysis. Therefore, the sample size is equal the sample size  $n$ . We use two approaches here: (1) we extract information as network statistics from a network, and (2) we extract information through a latent space model.

### Use network statistics

We denote a network through a square adjacency matrix  $\mathbf{M}=[m_{ij}]$  with each  $m_{ij}$  denoting the connection between subject  $i$  and subject  $j$ . Based on the adjacency matrix, many node-based network statistics can be defined. For example, the statistic *degree* is a centrality measure that simply counts how many subjects a subject connects to in the network. The statistic *betweenness* measures the extent to which a subject lies on the paths between other subjects. Subjects with high betweenness influence how the information flows in the network. Both degree and betweenness quantify the importance of a subject in a network. For example, for our friendship network, if a student has a larger degree, he or she is more popular in the network. From a network, we can derive a vector of network statistics for each subject  $i$  as  $\mathbf{t}_i(\mathbf{M})$ .

Because the network statistics are node based, the dimension of the resulting network statistics data will match the non-network data, and they can be combined to be used in SEM as any regular SEM analysis.

### Use latent space model

In this approach, each subject assumes a position in a Euclidean space. The distance of two subjects in the latent space is assumed to be related to how likely they are connected in the network. The idea of latent space modeling is similar to that of factor analysis with a latent factor space and factor scores. Let  $\mathbf{z}_i$  be a vector of latent positions of subject  $i$  in the latent space. For subjects  $i$  and  $j$ , the Euclidean distance between them is:

$$d_{ij}(\mathbf{z}_i, \mathbf{z}_j) = \sqrt{(\mathbf{z}_i - \mathbf{z}_j)^T (\mathbf{z}_i - \mathbf{z}_j)} = \sqrt{\sum_{d=1}^D (z_{i,d} - z_{j,d})^2}$$

\label{eq:distance}

where  $(\cdot)^T$  is the transpose of a matrix or vector,  $D$  is the dimension of the Euclidean latent space,  $\mathbf{z}_i = (z_{i,1}, z_{i,2}, \dots, z_{i,D})^T$  and  $\mathbf{z}_j = (z_{j,1}, z_{j,2}, \dots, z_{j,D})^T$  are the latent positions of subjects  $i$  and  $j$ , respectively. With the distance, the latent space model can be written as

$$\begin{cases} m_{ij} \sim \text{Bernoulli}(p_{ij}) \\ \text{logit}[p(m_{ij})] = \alpha + \boldsymbol{\beta}' \mathbf{h}_{ij} - \kappa d_{ij}(\mathbf{z}_i, \mathbf{z}_j) \end{cases}$$

\label{eq:LSM}

where  $\alpha$  is an intercept,  $\mathbf{h}_{ij}$  is a vector of covariates and  $\boldsymbol{\beta}$  contains the coefficients of the covariates. Note that the network is assumed to be unweighted here. In our software, following the tradition in network analysis, the coefficient  $\kappa$  for  $d_{ij}$  is fixed as 1 because  $\kappa$  can be rescaled together with the distance (Hoff et al., 2002). Therefore, the closer of two subjects are in the latent space, the higher the probability is for them to be connected after controlling the covariates in the model.

Here, we adapt and extend the latent space model to have the form shown below:

$$\begin{cases} E(m_{ij}) = \mu_{ij} \\ g(\mu_{ij}) = \alpha - d_{ij}(\mathbf{z}_i, \mathbf{z}_j) \end{cases}$$

\label{eq:SEM-LSM}

where  $g$  is a link function. First, we assume the connection between two subjects is solely explained by the latent space. Second, we relax the requirement of the Bernoulli distribution to use any exponential family of distributions. Using this model, we can extract information from a network. The idea is similar to principal component analysis. In our model, the latent positions will be used along with non-network variables in the SEM framework.

# Network edges as analysis units

Another approach we take is to use edges as the unit of interest. In this case, non-network data are reformatted for analysis to be based on pairs of individuals. In this case, given a non-network covariate  $c$ , we define  $c_{ij} = f(c_i, c_j)$ , where  $c_i$  and  $c_j$  are the covariate values for individual  $i$  and individual  $j$ . The function  $f$  can be chosen according to the purpose of the analysis. For example,  $c_{ij}$  can be the average of  $c_i$  and  $c_j$ , or it can be the difference. Then, these pairwise non-network variables can be used as either endogenous or exogenous variables.

## Use network statistics

Similar as in the node-based framework, in the edge-based framework, network statistics that can be obtained free from assuming underlying models to the social network can be used in SEM. The network statistics are constructed based on each pairs of subjects. For example, the shortest path length between each pair of nodes can be used as the edge-based network statistics.

## Use latent space model

The latent space modeling approach can also be used when using a pair of subjects as the unit of analysis. In this case, the latent distance between two subjects  $d_{ij}(z_i, z_j)$  can be used in SEM instead of the latent positions  $z_i$  and  $z_j$ .

# Example datasets

We will use several datasets to illustrate the use of our software.

## Friendship Network Data

In this dataset, information on friendship network, alcohol use, smoking, the big five personality traits, and academic performance among college students is collected for three years in 2017, 2018, and 2019. The participants were undergraduate students and the sample size is  $N = 165$ . There were about an equal number of male and female students (45% vs. 55%) in the sample. The average age of the students was 21.64 ( $SD = 0.85$ ). The average GPA of the students was about 3.273 ( $SD = 0.53$ ) out of 5.

Information on two social networks was collected. First, each student was presented a list of all the students in the study and was asked to report his/her acquaintanceship with everyone else on the list, on a Likert scale of 0 to 4. Second, each student was asked to report whether the students on the list were their WeChat friends or not (WeChat is a popular social network platform in China). Therefore, there are two friendship networks: the first one is a real-life weighted acquaintanceship network (referred to as the acquaintance network) and the second one is a virtual unweighted social media network (referred to as the WeChat network). The two networks together can be viewed as a multiplex network. Data on personality, happiness, depression, and loneliness were also collected.

## Attorney Network Data

The second dataset includes the cowork and advice network dataset from 71 attorneys from a law firm called SG&R in 1988. The dataset is available from the [SIENA website](#). The first wave of network data will be used in the analysis in the current tutorial. The cowork information is collected by asking the company employees to select people who have worked on the same case with them. Additionally, information on an advice network is collected via asking respondents who they seek advice from at work. Several non-network attributes are collected alongside with the networks. From those, the office one works at (i.e., Boston, Hartford, and Providence) and years with the firm will be used for analysis.

## Florentine Marriage Data

The dataset is from Breiger and Pattison (1986), where the social network indicates marriage alliances, and the non-network variables include (1) wealth, each family's net wealth in 1427 (in

thousands of lira); (2) priorates, the number of priorates (seats on the civic council) held between 1282- 1344; and (3) totalties, the total number of business or marriage ties in the total dataset of 116 families.

# Node based analysis with network statistics

The function `sem.net` can be used to fit a SEM model with network data using node statistics as variables. User-specified network statistics will be calculated and used as variables instead of the networks themselves in the SEM.

The following choices of network statistics can be used:

- `degree`: Degree is a centrality measure that counts actors/nodes a specific node is connected to.
- `betweenness`: Betweenness is a centrality measure that counts how many shortest path an actor is crossed by through a random choice. It measures how much an individual control the spread of information.
- `closeness`: Closeness is a measure of how efficiently a node spreads information and can be calculated by the average inverse distance from a node to all other nodes.
- `evcent`: The eigenvector centrality is a measure of transitive influence of each node, meaning that a node with high eigenvector centrality tends to connect with other nodes with high eigenvector centrality (Ruhnau, 2000).
- `stresscent`: Stress centrality is similar to betweenness centrality as it also measures the control of spread. However, while betweenness centrality measures through a random fraction of shortest paths, stress centrality takes into account all shortest paths (Szczepanski et al., 2012).
- `infocent`: Information centrality is defined as the reduction in network efficiency if a target node is removed. It is a measure of node effectiveness in spreading information (Latora and Marchiori, 2007).
- `ivi`: Integrated value of influence is a measure that combines different centrality measures (Salavaty et al., 2020a)
- `hubeness.score`: Hubeness score is a component of IVI and measures a node's influence in its surrounding environment.
- `spreading.score`: Spreading score is another component of IVI and measures a node's spreading potential.
- `clusterRank`: Cluster rank is a measure of clustering that takes into account a node, its neighbors, and their clustering coefficients.

## Simulated Data Example

To begin with, a random simulated dataset can be used to demonstrate the usage of the node-based network statistics approach. The code below generate a simulated network `net` with four non-network covariates `x1 - x4` which loads on two latent variables `lv1, lv2`.

```

set.seed(100)
nsamp = 100 # sample size
net <- ifelse(matrix(rnorm(nsamp^2), nsamp, nsamp) > 1, 1, 0) # simulate network
mean(net) # density of simulated network

# simulate non-network variables
lv1 <- rnorm(nsamp)
lv2 <- rnorm(nsamp)
nonnet <- data.frame(x1 = lv1*0.5 + rnorm(nsamp),
                    x2 = lv1*0.8 + rnorm(nsamp),
                    x3 = lv2*0.5 + rnorm(nsamp),
                    x4 = lv2*0.8 + rnorm(nsamp))

```

With the simulated data, we can define a `model` string with lavaan syntax that specifies the measurement model as well as the relationship between the network and the non-network variables. In this case, we are using `net` as a mediator between the two latent variables. Since data are generated randomly, the effects should be small overall.

```

model <- '
  lv1 =~ x1 + x2
  lv2 =~ x3 + x4
  net ~ lv2
  lv1 ~ net + lv2
'

```

Arguments passed to the `sem.net` function includes the model, the dataset, and the network statistics of interest. Note that `data` here should be a list with two elements, one being the named list of all network variables and one being the dataframe containing non-network variables. A `summary` function can be used to look at the output, and the function `path.networksem` can be used to look at mediation effects.

```

data = list(network = list(net = net), nonnetwork = nonnet)
set.seed(100)
res <- sem.net(model = model, data = data, netstats = c('degree'))
summary(res)
path.networksem(res, "lv2", c("net.degree"), "lv1")

```

The output of should look like the following.

```

> summary(res)
The SEM output:

```



lavaan 0.6.15 ended normally after 54 iterations

Estimator	ML
Optimization method	NLMINB
Number of model parameters	12
Number of observations	100

Model Test User Model:

Test statistic	1.230
Degrees of freedom	3
P-value (Chi-square)	0.746

Model Test Baseline Model:

Test statistic	24.987
Degrees of freedom	10
P-value	0.005

User Model versus Baseline Model:

Comparative Fit Index (CFI)	1.000
Tucker-Lewis Index (TLI)	1.394

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-913.294
Loglikelihood unrestricted model (H1)	-912.679
Akaike (AIC)	1850.588
Bayesian (BIC)	1881.850
Sample-size adjusted Bayesian (SABIC)	1843.951

Root Mean Square Error of Approximation:

RMSEA	0.000
90 Percent confidence interval - lower	0.000
90 Percent confidence interval - upper	0.118
P-value H_0: RMSEA <= 0.050	0.810

P-value H<sub>0</sub>: RMSEA >= 0.080 0.120

Standardized Root Mean Square Residual:

SRMR 0.026

Parameter Estimates:

Standard errors	Standard
Information	Expected
Information saturated (h1) model	Structured

Latent Variables:

	Estimate	Std.Err	z-value	P(> z )
lv2 =~				
x4	1.000			
x3	2.035	2.162	0.941	0.347
lv1 =~				
x2	1.000			
x1	1.056	0.789	1.338	0.181

Regressions:

	Estimate	Std.Err	z-value	P(> z )
lv1 ~				
lv2	-0.441	0.300	-1.470	0.142
net.degree ~				
lv2	-0.934	1.163	-0.804	0.422
lv1 ~				
net.degree	-0.011	0.020	-0.569	0.569

Variances:

	Estimate	Std.Err	z-value	P(> z )
.x4	1.350	0.293	4.603	0.000
.x3	0.215	0.923	0.233	0.816
.x2	1.002	0.299	3.357	0.001
.x1	1.047	0.328	3.190	0.001
.net.degree	22.292	3.164	7.046	0.000
lv2	0.214	0.249	0.860	0.390
.lv1	0.302	0.264	1.142	0.253

```
> path.networksem(res, "lv2", c("net.degree"), "lv1")
predictor mediator outcome apath bpath indirect indirect_se indirect_z
1 lv2 net.degree lv1 -0.934393 -0.01126621 0.01052707 1.086552 0.009688509
```

## Empirical Data Example

Using the friendship network data, a model with 5 personality traits and two networks' effect on happiness can be fitted using the code below. In this case, degree, betweenness, closeness are used as network statistics.

```
# load data
load("data/cf_data_book.RData") ## load the list cf_data

## data - non-network variables
non_network <- as.data.frame(cf_data$cf_nodal_cov)
dim(non_network)

## network - network variables (friends network and wechat network)
## note that the names of the networks are used in model specification
network <- list()
network$friends <- cf_data$cf_friend_network
network$wechat <- cf_data$cf_wwechat_network

model <- '
  Extroversion =~ personality1 + personality6
               + personality11 + personality16
  Conscientiousness =~ personality2 + personality7
                   + personality12 + personality17
  Neuroticism =~ personality3 + personality8
              + personality13 + personality18
  Openness =~ personality4 + personality9
            + personality14 + personality19
  Agreeableness =~ personality5 + personality10 +
                personality15 + personality20
  Happiness =~ happy1 + happy2 + happy3 + happy4
  friends ~ Extroversion + Conscientiousness + Neuroticism +
            Openness + Agreeableness
  Happiness ~ friends + wechat
'
```

```
## run sem.net
data = list(
  nonnetwork = non_network,
  network = network
)

set.seed(100)
res <- sem.net(model=model, data=data,
  netstats=c("degree", "betweenness", "closeness"),
  netstats.rescale = T,
  netstats.options=list("degree"=list("cmode"="freeman")))

## results
summary(res)
```

The output of the analysis is given below:

lavaan 0.6-18 ended normally after 453 iterations

Estimator	ML
Optimization method	NLMINB
Number of model parameters	82

Number of observations	165
------------------------	-----

Model Test User Model:

Test statistic	844.769
Degrees of freedom	377
P-value (Chi-square)	0.000

Model Test Baseline Model:

Test statistic	1795.826
Degrees of freedom	432
P-value	0.000

User Model versus Baseline Model:

Comparative Fit Index (CFI)	0.657
Tucker-Lewis Index (TLI)	0.607

#### Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-6286.542
Loglikelihood unrestricted model (H1)	-5864.157

Akaike (AIC)	12737.084
Bayesian (BIC)	12991.771
Sample-size adjusted Bayesian (SABIC)	12732.159

#### Root Mean Square Error of Approximation:

RMSEA	0.087
90 Percent confidence interval - lower	0.079
90 Percent confidence interval - upper	0.095
P-value H <sub>0</sub> : RMSEA ≤ 0.050	0.000
P-value H <sub>0</sub> : RMSEA ≥ 0.080	0.922

#### Standardized Root Mean Square Residual:

SRMR	0.116
------	-------

#### Parameter Estimates:

Standard errors	Standard
Information	Expected
Information saturated (h1) model	Structured

#### Latent Variables:

	Estimate	Std.Err	z-value	P(> z )
Happiness =~				
happy4	1.000			
happy3	-4.283	3.684	-1.162	0.245
happy2	-6.682	5.698	-1.173	0.241
happy1	-6.955	5.932	-1.172	0.241
Agreeableness =~				
personality20	1.000			
personality15	-1.200	0.905	-1.326	0.185

personality10	-4.293	2.506	-1.713	0.087
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personality5	-4.462	2.606	-1.712	0.087
--------------	--------	-------	--------	-------

Openness =~

personality19	1.000
---------------	-------

personality14	0.784	0.165	4.748	0.000
---------------	-------	-------	-------	-------

personality9	-0.224	0.106	-2.110	0.035
--------------	--------	-------	--------	-------

personality4	-0.097	0.108	-0.898	0.369
--------------	--------	-------	--------	-------

Neuroticism =~

personality18	1.000
---------------	-------

personality13	-0.532	0.148	-3.603	0.000
---------------	--------	-------	--------	-------

personality8	-0.808	0.176	-4.602	0.000
--------------	--------	-------	--------	-------

personality3	-0.378	0.136	-2.778	0.005
--------------	--------	-------	--------	-------

Conscientiousness =~

personality17	1.000
---------------	-------

personality12	-0.693	0.214	-3.235	0.001
---------------	--------	-------	--------	-------

personality7	-0.508	0.219	-2.319	0.020
--------------	--------	-------	--------	-------

personality2	1.108	0.265	4.187	0.000
--------------	-------	-------	-------	-------

Extroversion =~

personality16	1.000
---------------	-------

personality11	0.609	0.136	4.493	0.000
---------------	-------	-------	-------	-------

personality6	-0.508	0.123	-4.116	0.000
--------------	--------	-------	--------	-------

personality1	-0.521	0.119	-4.377	0.000
--------------	--------	-------	--------	-------

Regressions:

	Estimate	Std.Err	z-value	P(> z )
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friends.degree ~

Extroversion	2.355	1.126	2.091	0.037
--------------	-------	-------	-------	-------

friends.betweenness ~

Extroversion	2.119	1.048	2.023	0.043
--------------	-------	-------	-------	-------

friends.closeness ~

Extroversion	2.175	1.026	2.119	0.034
--------------	-------	-------	-------	-------

friends.degree ~

Conscientisnss	-8.447	5.060	-1.670	0.095
----------------	--------	-------	--------	-------

friends.betweenness ~

Conscientisnss	-7.827	4.706	-1.663	0.096
----------------	--------	-------	--------	-------

friends.closeness ~

Conscientisnss	-7.720	4.609	-1.675	0.094
----------------	--------	-------	--------	-------

friends.degree ~

Neuroticism	-1.282	1.364	-0.940	0.347
-------------	--------	-------	--------	-------

friends.betweenness ~

Neuroticism	-1.252	1.272	-0.985	0.325
friends.closeness ~				
Neuroticism	-1.324	1.248	-1.061	0.289
friends.degree ~				
Openness	-1.355	1.483	-0.914	0.361
friends.betweenness ~				
Openness	-1.204	1.377	-0.875	0.382
friends.closeness ~				
Openness	-1.162	1.348	-0.862	0.389
friends.degree ~				
Agreeableness	-16.541	15.253	-1.084	0.278
friends.betweenness ~				
Agreeableness	-15.697	14.299	-1.098	0.272
friends.closeness ~				
Agreeableness	-14.400	13.668	-1.054	0.292
Happiness ~				
friends.degree	-0.047	0.051	-0.931	0.352
frinds.btwnnss	0.007	0.025	0.292	0.771
friends.clsnss	0.062	0.059	1.045	0.296
wechat.degree	0.013	0.037	0.351	0.725
wechat.btwnnss	0.050	0.049	1.027	0.305
wechat.closnss	-0.064	0.060	-1.063	0.288

#### Covariances:

	Estimate	Std.Err	z-value	P(> z )
Agreeableness ~~				
Openness	0.015	0.018	0.866	0.386
Neuroticism	0.043	0.029	1.479	0.139
Conscientisnss	-0.072	0.044	-1.643	0.100
Extroversion	-0.011	0.020	-0.554	0.579
Openness ~~				
Neuroticism	0.330	0.074	4.446	0.000
Conscientisnss	-0.166	0.059	-2.806	0.005
Extroversion	0.089	0.080	1.111	0.266
Neuroticism ~~				
Conscientisnss	-0.153	0.058	-2.648	0.008
Extroversion	0.212	0.082	2.588	0.010
Conscientiousness ~~				
Extroversion	0.174	0.070	2.490	0.013

Variances:

	Estimate	Std.Err	z-value	P(> z )
.happy4	2.702	0.298	9.066	0.000
.happy3	1.226	0.147	8.353	0.000
.happy2	0.577	0.139	4.146	0.000
.happy1	0.507	0.145	3.496	0.000
.personality20	1.107	0.123	8.979	0.000
.personality15	1.195	0.134	8.945	0.000
.personality10	0.617	0.115	5.359	0.000
.personality5	0.742	0.130	5.705	0.000
.personality19	0.244	0.125	1.948	0.051
.personality14	0.680	0.107	6.372	0.000
.personality9	0.854	0.095	8.982	0.000
.personality4	0.963	0.106	9.067	0.000
.personality18	0.498	0.104	4.790	0.000
.personality13	0.920	0.109	8.469	0.000
.personality8	0.965	0.125	7.694	0.000
.personality3	0.893	0.102	8.768	0.000
.personality17	0.707	0.088	8.051	0.000
.personality12	1.042	0.119	8.753	0.000
.personality7	1.286	0.144	8.940	0.000
.personality2	1.193	0.143	8.337	0.000
.personality16	0.595	0.152	3.917	0.000
.personality11	1.125	0.140	8.023	0.000
.personality6	1.043	0.126	8.305	0.000
.personality1	0.902	0.111	8.122	0.000
.friends.degree	0.074	0.026	2.872	0.004
.frinds.btwnnss	0.236	0.034	6.912	0.000
.friends.clsnss	0.170	0.029	5.849	0.000
.Happiness	0.024	0.040	0.587	0.557
Agreeableness	0.030	0.034	0.874	0.382
Openness	0.652	0.155	4.209	0.000
Neuroticism	0.495	0.129	3.822	0.000
Conscientisnss	0.248	0.082	3.038	0.002
Extroversion	0.843	0.199	4.240	0.000

The multiple mediation from Agreeableness to friendship network to Happiness can be calculated using the following code.



```

> path.networksem(res, 'Agreeableness',
                  c('friends.degree', 'friends.betweenness', 'friends.closeness'),
                  'Happiness')

      predictor      mediator outcome   apath    bpath indirect
1 Agreeableness  friends.degree Happiness -16.54130 -0.047133471 0.7796491
2 Agreeableness friends.betweenness Happiness -15.69767 0.007403778 -0.1162220
3 Agreeableness  friends.closeness Happiness -14.40081 0.061957757 -0.8922416
indirect_se indirect_z
1 252.3110 0.0030900323
2 224.4727 -0.0005177557
3 196.8378 -0.0045328765

```

The model used here is shown in the diagram below. The model has the following features:

- We use two networks - friendship and WeChat networks.
- Three network statistics are used - degree, closeness, and betweenness.
- Friendship network is used as mediators.



# Node based analysis with latent space model

The node-based latent space model approach calculates latent positions of the networks, and use them in the SEM analysis along with non-network variables.

## Simulated Data Example

To begin with, a random simulated dataset can be used to demonstrate the usage of the node-based network statistics approach. The code below generate a simulated network `net` with four non-network covariates `x1 - x4` which loads on two latent variables `lv1, lv2`.

```
set.seed(10)
nsamp = 50
net <- ifelse(matrix(rnorm(nsamp^2), nsamp, nsamp) > 1, 1, 0)
mean(net) # density of simulated network
lv1 <- rnorm(nsamp)
lv2 <- rnorm(nsamp)
nonnet <- data.frame(x1 = lv1*0.5 + rnorm(nsamp),
                     x2 = lv1*0.8 + rnorm(nsamp),
                     x3 = lv2*0.5 + rnorm(nsamp),
                     x4 = lv2*0.8 + rnorm(nsamp))
```

With the simulated data, we can define a `model` string with lavaan syntax that specifies the measurement model as well as the relationship between the network and the non-network variables. In this case, we are using `net` as a mediator between the two latent variables. Since data are generated randomly, the effects should be small overall.

```
model <- '
  lv1 =~ x1 + x2
  lv2 =~ x3 + x4
  net ~ lv2
  lv1 ~ net + lv2
'
```

Arguments passed to the `sem.net.lsm` function includes the model, the dataset, and the number of latent dimensions. Note that `data` here should be a list with two elements, one being the named

list of all network variables and one being the dataframe containing non-network variables. A `summary` function can be used to look at the output, and the function `path.networksem` can be used to look at mediation effects across the two latent dimensions.

```
data = list(network = list(net = net), nonnetwork = nonnet)
set.seed(100)
res <- sem.net.lsm(model = model, data = data, latent.dim = 2)
summary(res)
path.networksem(res, 'lv2', c('net.Z1', 'net.Z2'), 'lv1')
```

The output looks like the following.

```
> summary(res)
Model Fit InformationSEM Test statistics: 3.771276 on 6 df with p-value: 0.7075962
NOTE: It is not certain whether it is appropriate to use latentnet's BIC to select latent space dimension, whether
or not to include actor-specific random effects, and to compare clustered models with the unclustered model.
network 1 LSM BIC: 2242.696
=====
=====

The SEM output:
lavaan 0.6.15 ended normally after 117 iterations

Estimator          ML
Optimization method  NLMINB
Number of model parameters      15

Number of observations      50

Model Test User Model:

Test statistic      3.771
Degrees of freedom      6
P-value (Chi-square)  0.708

Model Test Baseline Model:

Test statistic      34.438
Degrees of freedom      15
P-value              0.003
```

#### User Model versus Baseline Model:

Comparative Fit Index (CFI)	1.000
Tucker-Lewis Index (TLI)	1.287

#### Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-434.447
Loglikelihood unrestricted model (H1)	-432.561
Akaike (AIC)	898.893
Bayesian (BIC)	927.574
Sample-size adjusted Bayesian (SABIC)	880.491

#### Root Mean Square Error of Approximation:

RMSEA	0.000
90 Percent confidence interval - lower	0.000
90 Percent confidence interval - upper	0.138
P-value H <sub>0</sub> : RMSEA ≤ 0.050	0.765
P-value H <sub>0</sub> : RMSEA ≥ 0.080	0.165

#### Standardized Root Mean Square Residual:

SRMR	0.062
------	-------

#### Parameter Estimates:

Standard errors	Standard
Information	Expected
Information saturated (h1) model	Structured

#### Latent Variables:

	Estimate	Std.Err	z-value	P(> z )
lv2 =~				
x4	1.000			
x3	4.622	6.418	0.720	0.471
lv1 =~				
x2	1.000			

x1	-0.088	0.271	-0.326	0.744
----	--------	-------	--------	-------

Regressions:

	Estimate	Std.Err	z-value	P(> z )
lv1 ~				
lv2	-0.984	0.432	-2.279	0.023
net.Z1 ~				
lv2	-0.159	0.207	-0.765	0.444
net.Z2 ~				
lv2	0.208	0.257	0.809	0.418
lv1 ~				
net.Z1	-0.215	0.169	-1.277	0.202
net.Z2	0.255	0.138	1.850	0.064

Variances:

	Estimate	Std.Err	z-value	P(> z )
.x4	1.947	0.425	4.581	0.000
.x3	-1.587	3.655	-0.434	0.664
.x2	2.927	6.822	0.429	0.668
.x1	1.345	0.274	4.906	0.000
.net.Z1	0.624	0.124	5.012	0.000
.net.Z2	0.950	0.189	5.013	0.000
lv2	0.139	0.227	0.612	0.541
.lv1	-1.984	6.796	-0.292	0.770

The LSM output:

=====  
Summary of model fit  
=====

Formula: network::network(data\$network[[latent.network[i]]]) ~ euclidean(d = latent.dim)  
<environment: 0x7fc43202a550>

Attribute: edges

Model: Bernoulli

MCMC sample of size 4000, draws are 10 iterations apart, after burnin of 10000 iterations.

Covariate coefficients posterior means:

	Estimate	2.5%	97.5%	2*min(Pr(>0),Pr(<0))
(Intercept)	-0.18777	-0.42332	0.05	0.1175

Overall BIC: 2242.696  
Likelihood BIC: 2107.714  
Latent space/clustering BIC: 134.9814

Covariate coefficients MKL:

Estimate

(Intercept) -0.8639125

```
> path.networksem(res, 'lv2', c('net.Z1', 'net.Z2'), 'lv1')
predictor mediator outcome   apath   bpath indirect
1   lv2  net.Z1   lv1 -0.1587188 -0.2154100 0.03418961
2   lv2  net.Z2   lv1  0.2081154  0.2547222 0.05301162
indirect_se indirect_z
1 0.04030792 0.8482108
2 0.05368411 0.9874733
```

## Empirical Data Example

We fit the same model on the friendship and WeChat networks from the network statistics approach using the LSM approach. Under this approach, the latent positions take the roles of the network statistics but the model string can stay the same.

```
model <-'
Extroversion =~ personality1 + personality6
              + personality11 + personality16
Conscientiousness =~ personality2 + personality7
                  + personality12 + personality17
Neuroticism =~ personality3 + personality8
             + personality13 + personality18
Openness =~ personality4 + personality9
          + personality14 + personality19
Agreeableness =~ personality5 + personality10 +
               personality15 + personality20
Happiness =~ happy1 + happy2 + happy3 + happy4
friends ~ Extroversion + Conscientiousness + Neuroticism +
Openness + Agreeableness
Happiness ~ friends + wechat
'
```

To fit the model, the `sem.net.lsm()` function is used. The argument `latent.dim` should be used to denote the number of latent dimensions to be used in estimating the LSM. A random seed can be set to ensure reproduction of the results, and the argument `data.scale = T` is used so the scale of the latent positions and the non-network variables are not too different.

```
data = list(network=network, nonnetwork=non_network)
set.seed(100)
res <- sem.net.lsm(model=model,data=data, latent.dim = 2, data.rescale = T)
```

For SEM with latent positions, the estimation is again a two-stage process. First, a latent space model with no covariates is used to estimate latent positions through the `latentnet` R package. The resulting latent positions are then be extracted and compiled into the same dataset as the non-network variables such as the Big Five personality items and the happiness score items, which are then inputted into `lavaan` to be estimated in the SEM framework. We could again use `res$data` to access the restructured data with latent positions, and `res$model` to access the modified model string. The output of `sem.net.lsm()` has two components in `res$estimates` - `res$estimates$sem.es` for lavaan SEM results and `res$estimates$lsm.es` for latentnet LSM results.

The output of the analysis is given below:

```
> summary(res)
Model Fit InformationSEM Test statistics: 947.953 on 329 df with p-value: 0
network 1 LSM BIC: 15760.02
network 2 LSM BIC: 15517.77
=====
=====
```

The SEM output:

lavaan 0.6.15 ended normally after 147 iterations

Estimator	ML
Optimization method	NLMINB
Number of model parameters	74
Number of observations	165

Model Test User Model:

Test statistic	947.953
Degrees of freedom	329
P-value (Chi-square)	0.000



Model Test Baseline Model:

Test statistic	1448.277
Degrees of freedom	377
P-value	0.000

User Model versus Baseline Model:

Comparative Fit Index (CFI)	0.422
Tucker-Lewis Index (TLI)	0.338

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-5824.045
Loglikelihood unrestricted model (H1)	-5350.068
Akaike (AIC)	11796.089
Bayesian (BIC)	12025.929
Sample-size adjusted Bayesian (SABIC)	11791.645

Root Mean Square Error of Approximation:

RMSEA	0.107
90 Percent confidence interval - lower	0.099
90 Percent confidence interval - upper	0.115
P-value H <sub>0</sub> : RMSEA ≤ 0.050	0.000
P-value H <sub>0</sub> : RMSEA ≥ 0.080	1.000

Standardized Root Mean Square Residual:

SRMR	0.119
------	-------

Parameter Estimates:

Standard errors	Standard
Information	Expected
Information saturated (h1) model	Structured

Latent Variables:

Estimate	Std.Err	z-value	P(> z )
----------	---------	---------	---------

Happiness =~

happy4	1.000			
happy3	-5.462	4.485	-1.218	0.223
happy2	-8.435	6.866	-1.229	0.219
happy1	-8.634	7.029	-1.228	0.219

Agreeableness =~

personality20	1.000			
personality15	-0.915	0.722	-1.267	0.205
personality10	-4.359	2.395	-1.820	0.069
personality5	-3.726	2.043	-1.824	0.068

Openness =~

personality19	1.000			
personality14	0.658	0.144	4.571	0.000
personality9	-0.201	0.100	-2.004	0.045
personality4	-0.085	0.097	-0.873	0.383

Neuroticism =~

personality18	1.000			
personality13	-0.492	0.139	-3.529	0.000
personality8	-0.701	0.151	-4.651	0.000
personality3	-0.359	0.135	-2.664	0.008

Conscientiousness =~

personality17	1.000			
personality12	-0.475	0.163	-2.911	0.004
personality7	-0.383	0.159	-2.412	0.016
personality2	0.843	0.193	4.378	0.000

Extroversion =~

personality16	1.000			
personality11	0.632	0.151	4.181	0.000
personality6	-0.597	0.148	-4.038	0.000
personality1	-0.629	0.151	-4.170	0.000

Regressions:

Estimate Std.Err z-value P(>|z|)

friends.Z1 ~

Extroversion -0.150 0.179 -0.838 0.402

friends.Z2 ~

Extroversion -0.238 0.199 -1.192 0.233

friends.Z1 ~

Conscientisnss -0.047 0.327 -0.144 0.885

friends.Z2 ~

Conscientisnss	0.166	0.347	0.480	0.631
----------------	-------	-------	-------	-------

friends.Z1 ~

Neuroticism	-0.001	0.234	-0.006	0.995
-------------	--------	-------	--------	-------

friends.Z2 ~

Neuroticism	0.600	0.303	1.982	0.048
-------------	-------	-------	-------	-------

friends.Z1 ~

Openness	0.109	0.144	0.756	0.450
----------	-------	-------	-------	-------

friends.Z2 ~

Openness	-0.321	0.179	-1.794	0.073
----------	--------	-------	--------	-------

friends.Z1 ~

Agreeableness	0.335	1.023	0.328	0.743
---------------	-------	-------	-------	-------

friends.Z2 ~

Agreeableness	-0.957	1.176	-0.814	0.416
---------------	--------	-------	--------	-------

Happiness ~

friends.Z1	-0.029	0.025	-1.165	0.244
friends.Z2	-0.003	0.009	-0.394	0.693
wechat.Z1	0.027	0.024	1.146	0.252
wechat.Z2	-0.002	0.009	-0.192	0.848

#### Covariances:

	Estimate	Std.Err	z-value	P(> z )
Agreeableness ~~				
Openness	0.018	0.019	0.965	0.334
Neuroticism	0.041	0.027	1.538	0.124
Conscientisnss	-0.072	0.041	-1.727	0.084
Extroversion	-0.009	0.015	-0.553	0.580
Openness ~~				
Neuroticism	0.365	0.079	4.596	0.000
Conscientisnss	-0.152	0.068	-2.233	0.026
Extroversion	0.074	0.070	1.063	0.288
Neuroticism ~~				
Conscientisnss	-0.153	0.064	-2.391	0.017
Extroversion	0.177	0.068	2.605	0.009
Conscientiousness ~~				
Extroversion	0.130	0.063	2.073	0.038

#### Variances:

	Estimate	Std.Err	z-value	P(> z )
.happy4	0.985	0.109	9.065	0.000
.happy3	0.716	0.086	8.332	0.000

.happy2	0.332	0.080	4.141	0.000
.happy1	0.300	0.082	3.678	0.000
.personality20	0.965	0.108	8.968	0.000
.personality15	0.969	0.108	8.987	0.000
.personality10	0.436	0.116	3.773	0.000
.personality5	0.586	0.101	5.806	0.000
.personality19	0.205	0.154	1.326	0.185
.personality14	0.652	0.098	6.662	0.000
.personality9	0.962	0.107	9.013	0.000
.personality4	0.988	0.109	9.072	0.000
.personality18	0.485	0.105	4.635	0.000
.personality13	0.871	0.102	8.529	0.000
.personality8	0.744	0.096	7.720	0.000
.personality3	0.928	0.105	8.809	0.000
.personality17	0.591	0.106	5.555	0.000
.personality12	0.903	0.105	8.600	0.000
.personality7	0.935	0.106	8.781	0.000
.personality2	0.708	0.100	7.046	0.000
.personality16	0.443	0.116	3.831	0.000
.personality11	0.774	0.099	7.796	0.000
.personality6	0.797	0.100	7.983	0.000
.personality1	0.776	0.099	7.813	0.000
.friends.Z1	0.963	0.107	8.984	0.000
.friends.Z2	0.881	0.118	7.497	0.000
.Happiness	0.009	0.015	0.615	0.539
Agreeableness	0.029	0.031	0.934	0.350
Openness	0.789	0.186	4.234	0.000
Neuroticism	0.509	0.131	3.880	0.000
Conscientisnss	0.403	0.122	3.310	0.001
Extroversion	0.551	0.143	3.842	0.000

The LSM output:

=====

Summary of model fit

=====

Formula: network::network(data\$network[[latent.network[i]]]) ~ euclidean(d = latent.dim)

<environment: 0x7fc412d34470>

Attribute: edges

Model: Bernoulli

MCMC sample of size 4000, draws are 10 iterations apart, after burnin of 10000 iterations.

Covariate coefficients posterior means:

Estimate 2.5% 97.5% 2\*min(Pr(>0),Pr(<0))

(Intercept) 2.6130 2.5054 2.7225 < 2.2e-16 \*\*\*

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Overall BIC: 15760.02

Likelihood BIC: 14056.24

Latent space/clustering BIC: 1703.784

Covariate coefficients MKL:

Estimate

(Intercept) 2.426421

=====

Summary of model fit

=====

Formula: network::network(data\$network[[latent.network[i]]]) ~ euclidean(d = latent.dim)

<environment: 0x7fc412d34470>

Attribute: edges

Model: Bernoulli

MCMC sample of size 4000, draws are 10 iterations apart, after burnin of 10000 iterations.

Covariate coefficients posterior means:

Estimate 2.5% 97.5% 2\*min(Pr(>0),Pr(<0))

(Intercept) 1.1886 1.0938 1.2828 < 2.2e-16 \*\*\*

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Overall BIC: 15517.77

Likelihood BIC: 13970.87

Latent space/clustering BIC: 1546.901

Covariate coefficients MKL:

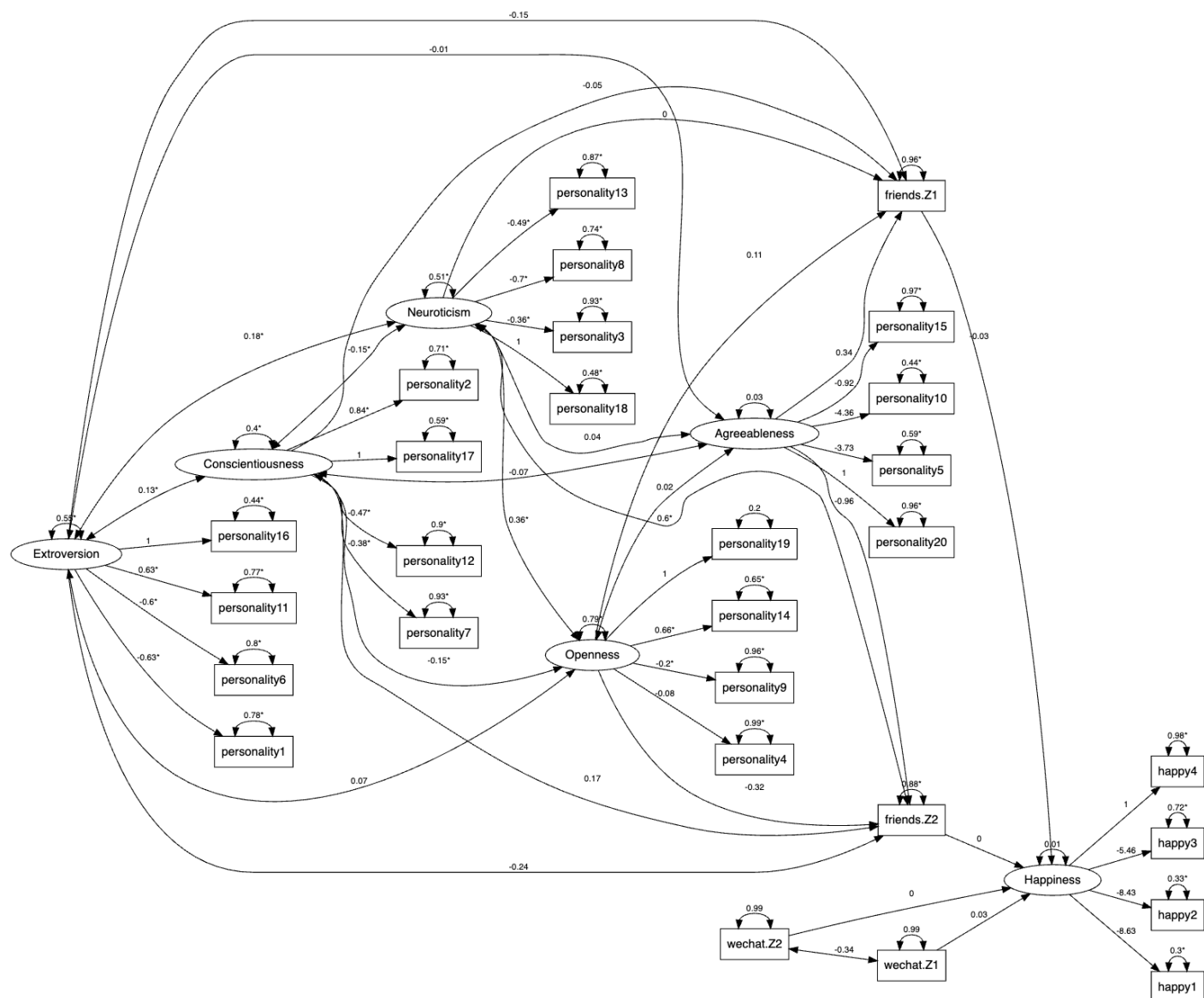
Estimate

(Intercept) 0.967353

The indirect effect from Agreeableness to the latent network positions then to Happiness is given below.

```
> path.networksem(res,
  'Agreeableness',
  c('friends.Z1', 'friends.Z2'),
  'Happiness')
predictor mediator outcome apath bpath
1 Agreeableness friends.Z1 Happiness 0.3354827 -0.028993008
2 Agreeableness friends.Z2 Happiness -0.9573035 -0.003419798
indirect indirect_se indirect_z
1 -0.009726651 0.343095 -0.028349729
2 0.003273785 1.125696 0.002908231
```

The path diagram is shown as the following.





# Edge based analysis with edge values

The edge based analysis can be conducted using the function `sem.net.edge`. The idea behind this method is that the edge values can be the unit of analysis if we transform non-network covariates into pair-based values.

## Simulated Data Example

To begin with, a random simulated dataset can be used to demonstrate the usage of the node-based network statistics approach. The code below generate a simulated network `net` with four non-network covariates `x1 - x4` which loads on two latent variables `lv1, lv2`.

```
set.seed(100)
nsamp = 100
net <- data.frame(ifelse(matrix(rnorm(nsamp^2), nsamp, nsamp) > 1, 1, 0))
mean(net) # density of simulated network
lv1 <- rnorm(nsamp)
lv2 <- rnorm(nsamp)
nonnet <- data.frame(x1 = lv1*0.5 + rnorm(nsamp),
                    x2 = lv1*0.8 + rnorm(nsamp),
                    x3 = lv2*0.5 + rnorm(nsamp),
                    x4 = lv2*0.8 + rnorm(nsamp))
```

With the simulated data, we can define a `model` string with lavaan syntax that specifies the measurement model as well as the relationship between the network and the non-network variables. In this case, we are using `net` as a mediator between the two latent variables. Since data are generated randomly, the effects should be small overall.

```
model <- '
  lv1 =~ x1 + x2
  lv2 =~ x3 + x4
  lv1 ~ net
  lv2 ~ lv1
'
```



Arguments passed to the `sem.net.edge` function includes the model and the dataset. Note that `data` here should be a list with two elements, one being the named list of all network variables and one being the dataframe containing non-network variables. A `summary` function can be used to look at the output, and the function `path.networksem` can be used to look at mediation effects.

```
data = list(network = list(net = net), nonnetwork = nonnet)
set.seed(100)
res <- sem.net.edge(model = model, data = data, type = 'difference')
summary(res)
path.networksem(res, "net", "lv1", "lv2")
```

The output is shown below.

```
> summary(res)
```

The SEM output:

lavaan 0.6.15 ended normally after 58 iterations

Estimator	ML
Optimization method	NLMINB
Number of model parameters	10
Number of observations	10000

Model Test User Model:

Test statistic	1.584
Degrees of freedom	4
P-value (Chi-square)	0.812

Model Test Baseline Model:

Test statistic	2296.506
Degrees of freedom	10
P-value	0.000

User Model versus Baseline Model:

Comparative Fit Index (CFI)	1.000
Tucker-Lewis Index (TLI)	1.003

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-75480.300
Loglikelihood unrestricted model (H1)	-75479.508
Akaike (AIC)	150980.601
Bayesian (BIC)	151052.704
Sample-size adjusted Bayesian (SABIC)	151020.925

Root Mean Square Error of Approximation:

RMSEA	0.000
90 Percent confidence interval - lower	0.000
90 Percent confidence interval - upper	0.009
P-value H_0: RMSEA <= 0.050	1.000
P-value H_0: RMSEA >= 0.080	0.000

Standardized Root Mean Square Residual:

SRMR	0.003
------	-------

Parameter Estimates:

Standard errors	Standard
Information	Expected
Information saturated (h1) model	Structured

Latent Variables:

	Estimate	Std.Err	z-value	P(> z )
lv1 =~				
x1	1.000			
x2	0.810	0.063	12.894	0.000
lv2 =~				
x3	1.000			
x4	0.302	0.056	5.377	0.000

Regressions:

	Estimate	Std.Err	z-value	P(> z )
lv1 ~				
net	0.053	0.039	1.371	0.170
lv2 ~				

```
lv1          -0.482  0.035 -13.683  0.000
```

Variances:

	Estimate	Std.Err	z-value	P(> z )
.x1	1.964	0.076	25.814	0.000
.x2	2.104	0.055	38.145	0.000
.x3	-0.681	0.527	-1.293	0.196
.x4	2.865	0.063	45.557	0.000
.lv1	0.898	0.077	11.708	0.000
.lv2	2.678	0.529	5.061	0.000

```
> path.networksem(res, "net", "lv1", "lv2")
predictor mediator outcome   apath   bpath indirect
1   net    lv1    lv2 0.05287153 -0.4823857 -0.02550447
indirect_se indirect_z
1 0.01705778 -1.495181
```

## Empirical Data Example

As an empirical example, we analyze the the attorney cowork and advice networks. In this example, the advice network is predicted by gender and years in practice, and the cowork network is predicted by the advice network, gender, and years in practice all together. In this case, the advice network acts as a mediator, while gender and years in practice exert indirect effect onto the cowork network through the advice network in addition to having direct effects. The model specification is given below.

```
non_network <- read.table("data/attorney/ELattr.dat")[,c(3,5)]
colnames(non_network) <- c('gender', 'years')
non_network$gender <- non_network$gender - 1
network <- list()
network$advice <- read.table("data/attorney/ELadv.dat")
network$cowork <- read.table("data/attorney/ELwork.dat")

model <- '
  advice ~ gender + years
  cowork ~ advice + gender + years
'
```

To use the function `sem.net.edge()`, we need to specify whether the covariate values to be run with the social network edge values in SEM should be calculated as the "difference" across two individuals or the "average" across two individuals. Here, the argument `ordered = c("cowork", "advice")`

is used to tell lavaan that the outcome variables cowork and advice are binary.

```
set.seed(100)
res <- sem.net.edge(model = model, data = data,
                    network = network, type = "difference", ordered = c("cowork", "advice"))
```

The output is shown as below.

lavaan 0.6.15 ended normally after 19 iterations

Estimator	DWLS
Optimization method	NLMINB
Number of model parameters	7

Number of observations	5041
------------------------	------

Model Test User Model:

	Standard	Scaled
Test Statistic	0.000	0.000
Degrees of freedom	0	0

Model Test Baseline Model:

Test statistic	1343.292	1343.292
Degrees of freedom	1	1
P-value	0.000	0.000
Scaling correction factor		1.000

User Model versus Baseline Model:

Comparative Fit Index (CFI)	1.000	1.000
Tucker-Lewis Index (TLI)	1.000	1.000
Robust Comparative Fit Index (CFI)		NA
Robust Tucker-Lewis Index (TLI)		NA

Root Mean Square Error of Approximation:

RMSEA	0.000	0.000
90 Percent confidence interval - lower	0.000	0.000

90 Percent confidence interval - upper	0.000	0.000
P-value H_0: RMSEA <= 0.050	NA	NA
P-value H_0: RMSEA >= 0.080	NA	NA

Robust RMSEA	NA	
90 Percent confidence interval - lower		NA
90 Percent confidence interval - upper		NA
P-value H_0: Robust RMSEA <= 0.050		NA
P-value H_0: Robust RMSEA >= 0.080		NA

Standardized Root Mean Square Residual:

SRMR	0.000	0.000
------	-------	-------

Parameter Estimates:

Standard errors	Robust.sem
Information	Expected
Information saturated (h1) model	Unstructured

Regressions:

	Estimate	Std.Err	z-value	P(> z )
advice ~				
gender	-0.019	0.040	-0.463	0.643
years	-0.018	0.002	-9.354	0.000
cowork ~				
advice	0.691	0.019	36.651	0.000
gender	0.013	0.040	0.323	0.747
years	0.013	0.002	7.248	0.000

Intercepts:

	Estimate	Std.Err	z-value	P(> z )
.advice	0.000			
.cowork	0.000			

Thresholds:

	Estimate	Std.Err	z-value	P(> z )
advice t1	0.956	0.022	43.812	0.000
cowork t1	1.037	0.022	48.049	0.000

Variances:

	Estimate	Std.Err	z-value	P(> z )
.advice	1.000			
.cowork	0.523			

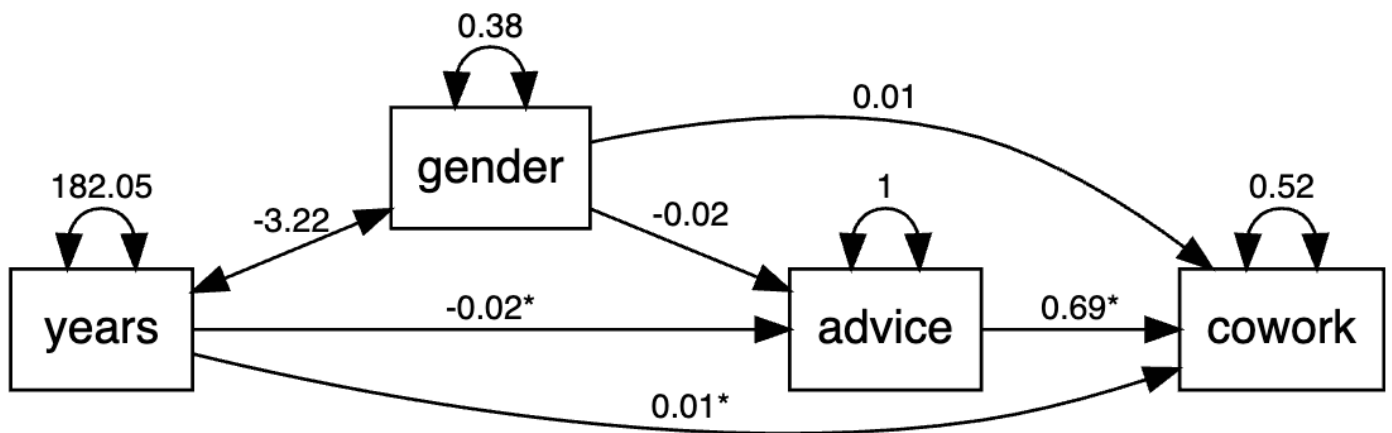
Scales y\*:

	Estimate	Std.Err	z-value	P(> z )
advice	1.000			
cowork	1.000			

The indirect effects can be calculated as below.

```
> path.networksem(res, "gender", "advice", "cowork")
predictor mediator outcome   apath   bpath indirect
1  gender  advice cowork -0.01856161 0.6909742 -0.01282559
  indirect_se indirect_z
1  0.01304666 -0.9830558
```

The model is shown in the graph below.



# Edge based analysis with latent space model

The R function `sem.net.edge.lsm` can be used to conduct edge based analysis with latent space model. In this case, the latent distance between each pair of individuals is used along with the transformed non-network covariates in SEM.

## Simulated Data Example

To begin with, a random simulated dataset can be used to demonstrate the usage of the node-based network statistics approach. The code below generate a simulated network `net` with four non-network covariates `x1 - x4` which loads on two latent variables `lv1, lv2`.

```
set.seed(10)
nsamp = 50
lv1 <- rnorm(nsamp)
net <- ifelse(matrix(rnorm(nsamp^2), nsamp, nsamp) > 1, 1, 0)
lv2 <- rnorm(nsamp)
nonnet <- data.frame(x1 = lv1*0.5 + rnorm(nsamp),
                    x2 = lv1*0.8 + rnorm(nsamp),
                    x3 = lv2*0.5 + rnorm(nsamp),
                    x4 = lv2*0.8 + rnorm(nsamp))
```

With the simulated data, we can define a `model` string with lavaan syntax that specifies the measurement model as well as the relationship between the network and the non-network variables. In this case, we are using `net` as a mediator between the two latent variables. Since data are generated randomly, the effects should be small overall.

```
model <- '
lv1 =~ x1 + x2
lv2 =~ x3 + x4
net ~ lv1
lv2 ~ net
'
```

Arguments passed to the `sem.net.edge.lsm` function includes the model, the dataset, and the latent dimensions. Note that `data` here should be a list with two elements, one being the named list of all

network variables and one being the dataframe containing non-network variables. A `summary` function can be used to look at the output.

```
data = list(network = list(net = net), nonnetwork = nonnet)
set.seed(100)
res <- sem.net.edge.lsm(model = model, data = data, latent.dim = 1)
summary(res)
path.networksem(res, 'lv2', c('net.dists'), 'lv1')
```

The output is shown below:

Model Fit InformationSEM Test statistics: 492.628 on 4 df with p-value: 0  
network 1 LSM BIC: 2244.546

=====

The SEM output:

lavaan 0.6.15 ended normally after 29 iterations

Estimator	ML
Optimization method	NLMINB
Number of model parameters	11
Number of observations	2500

Model Test User Model:

Test statistic	492.628
Degrees of freedom	4
P-value (Chi-square)	0.000

Model Test Baseline Model:

Test statistic	958.550
Degrees of freedom	10
P-value	0.000

User Model versus Baseline Model:

Comparative Fit Index (CFI)	0.485
-----------------------------	-------



Tucker-Lewis Index (TLI)	-0.288
--------------------------	--------

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-22209.465
-------------------------------	------------

Loglikelihood unrestricted model (H1)	NA
---------------------------------------	----

Akaike (AIC)	44440.930
--------------	-----------

Bayesian (BIC)	44504.994
----------------	-----------

Sample-size adjusted Bayesian (SABIC)	44470.045
---------------------------------------	-----------

Root Mean Square Error of Approximation:

RMSEA	0.221
-------	-------

90 Percent confidence interval - lower	0.205
--	-------

90 Percent confidence interval - upper	0.238
--	-------

P-value H_0: RMSEA <= 0.050	0.000
-----------------------------	-------

P-value H_0: RMSEA >= 0.080	1.000
-----------------------------	-------

Standardized Root Mean Square Residual:

SRMR	0.109
------	-------

Parameter Estimates:

Standard errors	Standard
Information	Expected
Information saturated (h1) model	Structured

Latent Variables:

	Estimate	Std.Err	z-value	P(> z )
lv2 =~				
x4	1.000			
x3	0.976	NA		
lv1 =~				
x2	1.000			
x1	0.642	NA		

Regressions:

Estimate	Std.Err	z-value	P(> z )
----------	---------	---------	---------

```

net.dists ~
  lv1      -0.000    NA
lv2 ~
  net.dists  -0.000    NA

```

Variances:

	Estimate	Std.Err	z-value	P(> z )
.x4	2.856	NA		
.x3	1.501	NA		
.x2	1.722	NA		
.x1	2.490	NA		
.net.dists	0.553	NA		
.lv2	1.315	NA		
lv1	0.715	NA		

The LSM output:

```

=====
Summary of model fit
=====

```

```

Formula: network::network(data$network[[latent.network[i]]]) ~ euclidean(d = latent.dim)
<environment: 0x7fc473af4960>

```

Attribute: edges

Model: Bernoulli

MCMC sample of size 4000, draws are 10 iterations apart, after burnin of 10000 iterations.

Covariate coefficients posterior means:

	Estimate	2.5%	97.5%	2*min(Pr(>0),Pr(<0))	
(Intercept)	-0.67923	-0.83587	-0.5504		< 2.2e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

```

Overall BIC:      2244.546
Likelihood BIC:   2184.507
Latent space/clustering BIC:  60.03918

```

Covariate coefficients MKL:

	Estimate
(Intercept)	-1.117408

# Empirical Data Example

When embedding the LSM into the edge-based approach, one thing that needs to be considered is whether to model covariates predicting the social networks in the LSM framework or in the SEM framework. This is only a concern in the edge-based model since covariates need to be edge-based as well if using the LSM method, and it defies the purpose of simplicity if we consider the LSM in the actor-based approach. In this example, we will accommodate the covariates in the LSM framework within the edge-based approach. The dataset used in this example is the Florentine marriage dataset. The model is quite simple as shown below. Essentially, the observed marriage network is hypothesized to be based not only on the latent positions, but also on the non-network variable of wealth. Additionally, priorates is viewed as a predictor of the distance between latent positions of the marriage networks.

```
load("data/flomarriage.RData")

network <- list()
network$flo <- flomarriage.network
nonnetwork <- flomarriage.nonnetwork

model <- '
  flo ~ wealth
  priorates ~ flo + wealth
'
```

When fitting the model using the `sem.net.edge.lsm` function, the argument `type` and `latent.dim` are needed. Here, although the marriage network contains binary edges, the ordered argument is not needed since only the continuous latent distances will be used in the SEM.

```
data = list(network=network, nonnetwork=nonnetwork)
set.seed(100)
res <- sem.net.edge.lsm(model=model,data=data, type = "difference", latent.dim = 2, netstats.rescale = T,
data.rescale = T)
## results
summary(res)
```

In this model, the `latentnet` package is first used to estimate the LSM with the covariate of wealth. Then, the resulting latent positions of the marriage network, taking apart the effect of wealth, is hypothesized to be influenced by priorates and the effect is estimated through `lavaan`. Thus, the latent distances of the marriage network acts like a mediator between priorates and the observed network. The resulting estimates from both the SEM component and the LSM component are shown below.

Model Fit InformationSEM Test statistics: 0 on 0 df with p-value: NA  
network 1 LSM BIC: 259.7975

=====

=====

The SEM output:

lavaan 0.6.15 ended normally after 6 iterations

Estimator	ML
Optimization method	NLMINB
Number of model parameters	5
Number of observations	256

Model Test User Model:

Test statistic	0.000
Degrees of freedom	0

Model Test Baseline Model:

Test statistic	50.126
Degrees of freedom	3
P-value	0.000

User Model versus Baseline Model:

Comparative Fit Index (CFI)	1.000
Tucker-Lewis Index (TLI)	1.000

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-700.431
Loglikelihood unrestricted model (H1)	-700.431
Akaike (AIC)	1410.863
Bayesian (BIC)	1428.589
Sample-size adjusted Bayesian (SABIC)	1412.737

Root Mean Square Error of Approximation:

RMSEA	0.000
90 Percent confidence interval - lower	0.000
90 Percent confidence interval - upper	0.000
P-value H_0: RMSEA <= 0.050	NA
P-value H_0: RMSEA >= 0.080	NA

Standardized Root Mean Square Residual:

SRMR	0.000
------	-------

Parameter Estimates:

Standard errors	Standard
Information	Expected
Information saturated (h1) model	Structured

Regressions:

	Estimate	Std.Err	z-value	P(> z )
priorates ~				
wealth	0.422	0.057	7.441	0.000
flo.dists ~				
wealth	0.000	0.063	0.000	1.000
priorates ~				
flo.dists	-0.000	0.057	-0.000	1.000

Variances:

	Estimate	Std.Err	z-value	P(> z )
.priorates	0.819	0.072	11.314	0.000
.flo.dists	0.996	0.088	11.314	0.000

The LSM output:

```
=====
Summary of model fit
=====
```

Formula: network::network(data\$network[[latent.network[i]]]) ~ euclidean(d = latent.dim)  
<environment: 0x7fc434ed5160>

Attribute: edges

Model: Bernoulli

MCMC sample of size 4000, draws are 10 iterations apart, after burnin of 10000 iterations.

Covariate coefficients posterior means:

Estimate 2.5% 97.5% 2\*min(Pr(>0),Pr(<0))

(Intercept) 5.0133 2.5627 7.9665 < 2.2e-16 \*\*\*

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Overall BIC: 259.7975

Likelihood BIC: 85.53086

Latent space/clustering BIC: 174.2666

Covariate coefficients MKL:

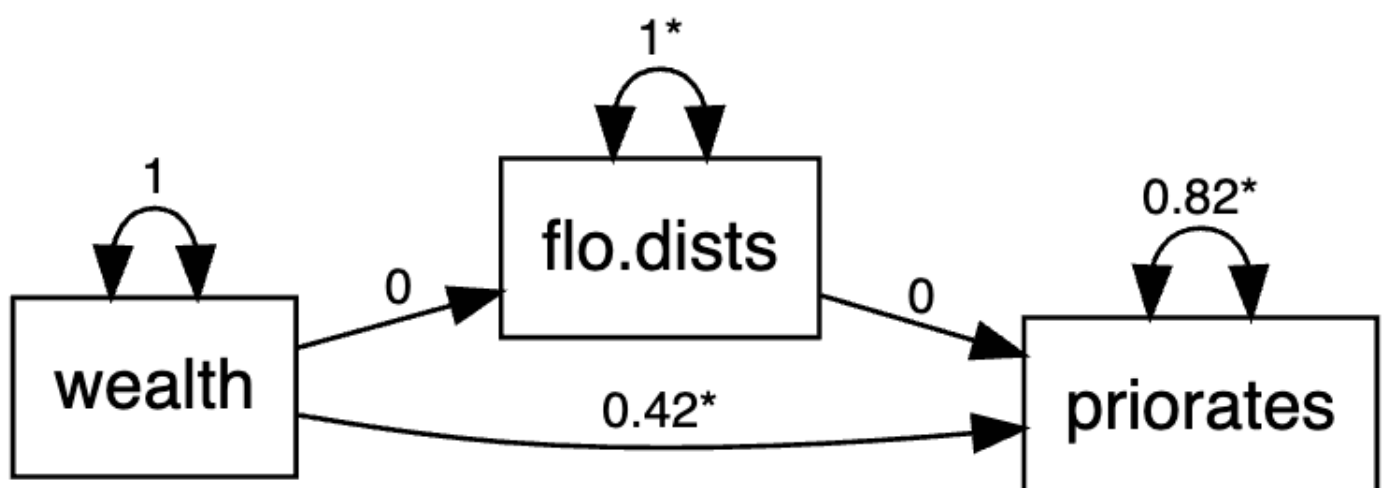
Estimate

(Intercept) 2.861026

To look at indirect effects, the following code can be used.

```
> path.networksem(res, "wealth", "flo.dists", "priorates")
predictor mediator outcome apath bpath indirect
1 wealth flo.dists priorates 2.976241e-21 -4.047181e-22 -1.204539e-42
indirect_se indirect_z
1 1.874237e-22 -6.42682e-21
```

The model is shown in this diagram below.



# Use of Web App for SEM with Networks

The network data analysis can also be conducted using our online app available at: <https://bigsem.psychstat.org/app> . To use the app, one need to register as a user to protect the data of the users. Once logging in, a user with work with an interface like below:

BigSEM

Welcome Johnny Zhang » Current Project | New Project | List All Projects | Apps | Q & A

Project: SEM-network

Path Diagram

Diagram It

Upload Files

New File

<input type="checkbox"/> File name	Operations	File Actions	File size	Time
<input type="checkbox"/> network.ex1.diag		<a href="#">Edit</a>   <a href="#">View</a>   <a href="#">Delete</a>   <a href="#">Download</a>   <a href="#">Rename</a>   <a href="#">History</a>	3.26 KB	2024.10.22 15:36:50.
<input type="checkbox"/> network.ex1.sem		<a href="#">Edit</a>   <a href="#">View</a>   <a href="#">Delete</a>   <a href="#">Download</a>   <a href="#">Rename</a>   <a href="#">History</a>	383 B	2024.10.22 15:36:50.
<input type="checkbox"/> network.ex1.sem.out		<a href="#">Edit</a>   <a href="#">View</a>   <a href="#">Delete</a>   <a href="#">Download</a>   <a href="#">Rename</a>   <a href="#">History</a>	9.4 KB	2024.10.22 15:36:50.
<input type="checkbox"/> network.ex2.net.edge.sem.out		<a href="#">Edit</a>   <a href="#">View</a>   <a href="#">Delete</a>   <a href="#">Download</a>   <a href="#">Rename</a>   <a href="#">History</a>	6.7 KB	2024.10.22 15:22:52.
<input type="checkbox"/> network.ex2.net.edge.diag		<a href="#">Edit</a>   <a href="#">View</a>   <a href="#">Delete</a>   <a href="#">Download</a>   <a href="#">Rename</a>   <a href="#">History</a>	3.27 KB	2024.10.22 15:01:48.
<input type="checkbox"/> network.ex2.net.edge.sem		<a href="#">Edit</a>   <a href="#">View</a>   <a href="#">Delete</a>   <a href="#">Download</a>   <a href="#">Rename</a>   <a href="#">History</a>	393 B	2024.10.22 15:01:48.
<input type="checkbox"/> network.RData		<a href="#">Edit</a>   <a href="#">View</a>   <a href="#">Delete</a>   <a href="#">Download</a>   <a href="#">Rename</a>   <a href="#">History</a>	17.81 KB	2024.10.21 21:41:08.
<input type="checkbox"/> cf_wechat.csv	<a href="#">Analysis</a>	<a href="#">Edit</a>   <a href="#">View</a>   <a href="#">Delete</a>   <a href="#">Download</a>   <a href="#">Rename</a>   <a href="#">History</a>	54.19 KB	2024.10.19 20:38:23.
<input type="checkbox"/> cf_friends.csv	<a href="#">Analysis</a>	<a href="#">Edit</a>   <a href="#">View</a>   <a href="#">Delete</a>   <a href="#">Download</a>   <a href="#">Rename</a>   <a href="#">History</a>	54.04 KB	2024.10.19 17:43:05.
<input type="checkbox"/> cf_nonnetwork.csv	<a href="#">Analysis</a>	<a href="#">Edit</a>   <a href="#">View</a>   <a href="#">Delete</a>   <a href="#">Download</a>   <a href="#">Rename</a>   <a href="#">History</a>	28.86 KB	2024.10.19 17:43:05.

Delete

Compare

## Organizing data

Organizing the data for analysis is the first step for using the app or R package. In R, the data are provided as a list with a non-network component and a network component. To conveniently organize the data online, we developed a simple app.

To use the app, one first upload the non-network data and network data sets as separate files. Then, in the app, one selects the corresponding data files. An example is given below with two

networks - friendship and WeChat networks. Note that the new data set will be saved as R data with the provided name, i.e., `mynetworkdata.RData` in this example.

## Organize network data

### Analysis Menu

New network data set name:

Non-network data:

Network data 1:

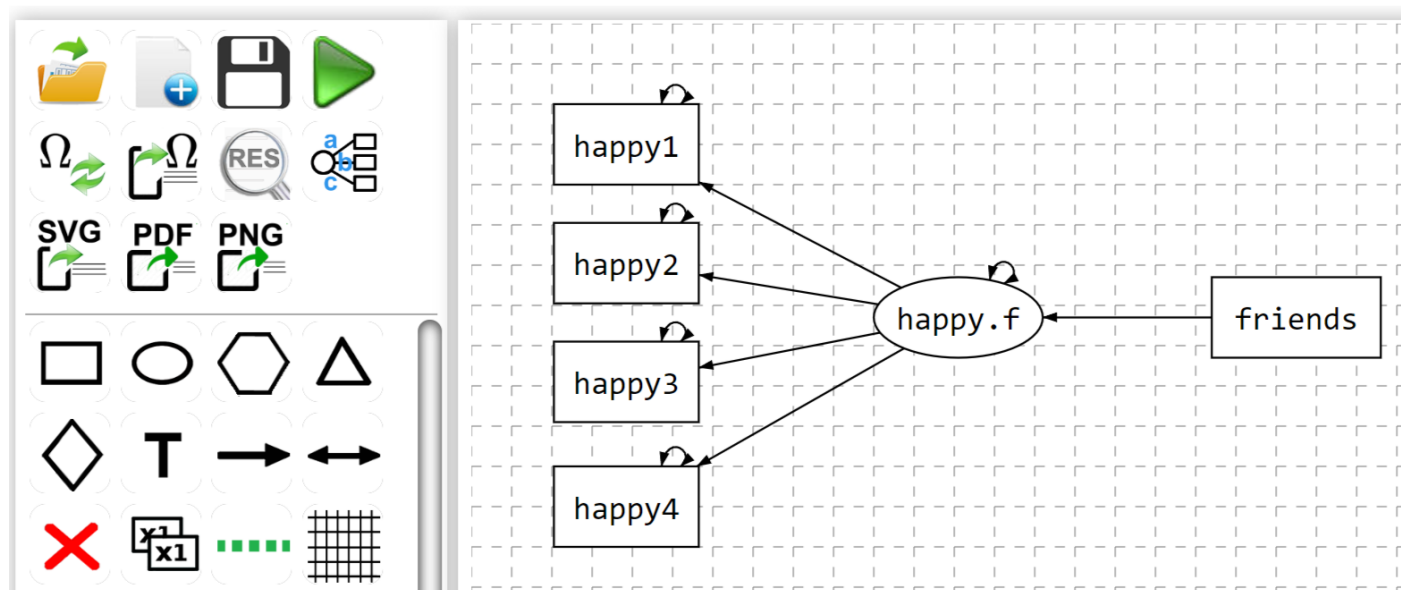
Network data 2:

Network data 3:

Network data 4:

## Conducting the analysis

We use a simple example to illustrate the use of the online app. To conduct the analysis, we need to first draw the path diagram of the model. Here, we create a latent happiness factor (`happy.f`) from the 4-item measure of global subjective happiness. We then use the friendship network to predict the happiness factor.





For the network analysis, one needs to choose the software to use, here "NetworkSEM". Then, one selects the Data File "network.RData".

## Software:

NetworkSEM ▾

## Data File:

network.RD ▾



For the network statistics based method, one need to choose what statistics to use. Here, one can specify them in the "Control" field. In this example, we use `netstats = degree, betweenness, closeness` to allow the use of the three network statistics.

## Control:

```
netstats=degree,  
betweenness, closeness
```

To run the analysis, one clicks on the green triangle in the left panel. The output of the analysis is given below. The output has several parts:

- The basic information, particularly, the user and the analysis id `7cf61d4792351966add082d56368301d`.
- The descriptive statistics for numerical variables in the non-network data set.
- The information on the networks.
- The basic model information
- The results from fitting the model.

BigSEM started at 15:36:50 on Oct 22, 2024.

=====

Please refresh your browser for complete output of complex data analysis.

The current analysis was conducted by the BigSEM user **johnny**.

To contact us, make sure to include the ticket # for this analysis

7cf61d4792351966add082d56368301d

## Descriptive statistics (N=165, p=59)

	Mean	sd	Min	Max	Skewness	Kurtosis
gender	0.55152	0.49885	0.000	1.0000	-0.2071631	1.0429
gpa	3.27293	0.48805	1.173	4.2200	-0.6399076	4.2619
age	21.64242	0.85505	18.000	24.0000	-0.1255522	4.5903
weight	62.29091	14.16756	37.000	110.0000	0.9021334	3.2265
height	169.54545	8.15808	155.000	188.0000	0.3186553	1.9660
smoke	0.26061	0.44030	0.000	1.0000	1.0907192	2.1897
drink	0.41212	0.49372	0.000	1.0000	0.3570735	1.1275
wechat	157.32927	180.36548	0.000	1000.0000	2.9199355	11.9943
id	83.00000	47.77552	1.000	165.0000	0.0000000	1.7999
personality1	2.81818	1.06652	1.000	5.0000	-0.0869982	2.4384
personality2	2.61818	1.22710	1.000	5.0000	0.3212422	2.0339
personality3	2.45455	0.98436	1.000	5.0000	0.4540597	2.8503
personality4	2.64242	0.98743	1.000	5.0000	0.1910639	2.5725
personality5	3.03636	1.15764	1.000	5.0000	-0.0235915	2.2242
personality6	3.07879	1.12612	1.000	5.0000	0.1017642	2.3871
personality7	3.27273	1.16537	1.000	5.0000	-0.1954555	2.1881
personality8	2.36970	1.13816	1.000	5.0000	0.5103888	2.4850
personality9	2.75758	0.94451	1.000	5.0000	0.3684034	3.1224
personality10	3.01212	1.08194	1.000	5.0000	0.0049198	2.5241
personality11	2.89697	1.20276	1.000	5.0000	0.0931915	2.2009
personality12	3.78788	1.08081	1.000	5.0000	-0.4433181	2.2537
personality13	2.61818	1.03283	1.000	5.0000	0.3473757	2.9438
personality14	3.80000	1.04298	1.000	5.0000	-0.5964333	2.8276
personality15	3.42424	1.11613	1.000	5.0000	-0.3898210	2.5711
personality16	2.65455	1.20292	1.000	5.0000	0.2450516	2.2534
personality17	2.31515	0.98033	1.000	5.0000	0.3493841	2.6210
personality18	3.59394	0.99937	1.000	5.0000	-0.1128832	2.1067
personality19	3.82424	0.94966	1.000	5.0000	-0.5435870	3.1673
personality20	3.12121	1.06946	1.000	5.0000	0.0874853	2.4055
depress1	0.98788	0.55202	0.000	3.0000	0.6478164	5.7357
depress2	0.61818	0.58926	0.000	3.0000	0.5205043	3.3723
depress3	0.76364	0.78002	0.000	3.0000	0.8239322	3.2396
depress4	0.91515	0.59884	0.000	3.0000	0.3722678	4.0971
depress5	0.70303	0.67376	0.000	3.0000	0.6728525	3.3429
depress6	0.80606	0.69753	0.000	3.0000	0.7141707	3.7965
depress7	0.66667	0.70998	0.000	3.0000	0.8848909	3.5949
lone1	1.04848	0.77935	0.000	3.0000	0.2260045	2.3813
lone2	1.26667	0.88437	0.000	3.0000	0.1437581	2.2374
lone3	1.03030	0.87251	0.000	3.0000	0.2729773	2.0401
lone4	1.29091	0.90404	0.000	3.0000	0.1403947	2.1952
lone5	1.27879	0.88750	0.000	3.0000	0.0558801	2.1521
lone6	0.85455	0.79828	0.000	3.0000	0.5543989	2.5604
lone7	0.98788	0.85531	0.000	3.0000	0.3749858	2.2210
lone8	1.64242	0.89682	0.000	3.0000	-0.2540419	2.3354
lone9	1.00000	0.86954	0.000	3.0000	0.3907138	2.2320
lone10	0.88485	0.76832	0.000	3.0000	0.5218129	2.7655
happy1	5.34545	1.31897	1.000	7.0000	-0.8142547	3.6334
happy2	5.25455	1.30969	1.000	7.0000	-0.7392627	3.2077
happy3	5.24848	1.30387	2.000	7.0000	-0.4342157	2.6097

happy4	3.89091	1.65654	1.000	7.0000	0.1177261	2.2404
lone	1.12848	0.56674	0.000	2.6000	-0.0868936	2.8135
depress	0.78009	0.41754	0.000	1.8571	0.1401042	2.5266
happy	4.93485	0.86774	2.500	7.0000	0.2112938	3.2653
p.e	2.91364	0.78605	1.000	5.0000	0.1731648	3.4108
p.c	3.53182	0.69743	2.000	5.0000	0.2454618	2.4799
p.i	3.53788	0.68721	1.500	5.0000	-0.2099051	2.6462
p.a	3.55606	0.61259	1.750	5.0000	0.0235716	2.8378
p.n	2.87576	0.63835	1.000	4.7500	0.1728206	3.3815
bmi	21.50942	3.84812	15.401	39.5197	1.5035276	6.1558

#### Missing Rate

gender	0.0000000
gpa	0.0000000
age	0.0000000
weight	0.0000000
height	0.0000000
smoke	0.0000000
drink	0.0000000
wechat	0.0060606
id	0.0000000
personality1	0.0000000
personality2	0.0000000
personality3	0.0000000
personality4	0.0000000
personality5	0.0000000
personality6	0.0000000
personality7	0.0000000
personality8	0.0000000
personality9	0.0000000
personality10	0.0000000
personality11	0.0000000
personality12	0.0000000
personality13	0.0000000
personality14	0.0000000
personality15	0.0000000
personality16	0.0000000
personality17	0.0000000
personality18	0.0000000
personality19	0.0000000
personality20	0.0000000
depress1	0.0000000
depress2	0.0000000
depress3	0.0000000
depress4	0.0000000
depress5	0.0000000
depress6	0.0000000
depress7	0.0000000
lone1	0.0000000
lone2	0.0000000
lone3	0.0000000
lone4	0.0000000
lone5	0.0000000
lone6	0.0000000

lone7	0.0000000
lone8	0.0000000
lone9	0.0000000
lone10	0.0000000
happy1	0.0000000
happy2	0.0000000
happy3	0.0000000
happy4	0.0000000
lone	0.0000000
depress	0.0000000
happy	0.0000000
p.e	0.0000000
p.c	0.0000000
p.i	0.0000000
p.a	0.0000000
p.n	0.0000000
bmi	0.0000000

## Network data information

	#row	#col
friends	165	165
wechat	165	165

## Model information

Observed non-network variables: happy1 happy2 happy3 happy4 .

Observed network variables: friends .

Latent variables: happy.f .

The weight is: 0 .

## Results

lavaan 0.6-18 ended normally after 66 iterations

Estimator	ML
Optimization method	NLMINB
Number of model parameters	11

Number of observations	165
------------------------	-----

Model Test User Model:

Test statistic	14.749
Degrees of freedom	11
P-value (Chi-square)	0.194

Model Test Baseline Model:

Test statistic	162.858
Degrees of freedom	18
P-value	0.000

User Model versus Baseline Model:

Comparative Fit Index (CFI)	0.974
Tucker-Lewis Index (TLI)	0.958

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-1077.697
Loglikelihood unrestricted model (H1)	-1070.322
Akaike (AIC)	2177.394
Bayesian (BIC)	2211.559
Sample-size adjusted Bayesian (SABIC)	2176.733

Root Mean Square Error of Approximation:

RMSEA	0.045
90 Percent confidence interval - lower	0.000
90 Percent confidence interval - upper	0.099
P-value H <sub>0</sub> : RMSEA ≤ 0.050	0.498
P-value H <sub>0</sub> : RMSEA ≥ 0.080	0.170

Standardized Root Mean Square Residual:

SRMR	0.039
------	-------

Parameter Estimates:

Standard errors	Standard
Information	Expected
Information saturated (h1) model	Structured

Latent Variables:

	Estimate	Std.Err	z-value	P(> z )
happy.f =~				
happy4	1.000			
happy3	-4.933	5.032	-0.980	0.327
happy2	-7.445	7.547	-0.986	0.324
happy1	-8.133	8.251	-0.986	0.324

Regressions:

	Estimate	Std.Err	z-value	P(> z )
happy.f ~				
friends.degree	-0.024	0.037	-0.655	0.513
frinds.btwnnss	0.019	0.029	0.654	0.513
friends.clsnss	0.011	0.027	0.401	0.689

Variances:

	Estimate	Std.Err	z-value	P(> z )
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.happy4	2.708	0.299	9.070	0.000
.happy3	1.219	0.147	8.306	0.000
.happy2	0.633	0.150	4.207	0.000
.happy1	0.450	0.167	2.701	0.007
.happy.f	0.019	0.039	0.494	0.621

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BigSEM ended at 15:36:50 on Oct 22, 2024