

# Edge based analysis with latent space model

The R function `sem.net.edge.lsm` can be used to conduct edge based analysis with latent space model. In this case, the latent distance between each pair of individuals is used along with the transformed non-network covariates in SEM.

## Simulated Data Example

To begin with, a random simulated dataset can be used to demonstrate the usage of the node-based network statistics approach. The code below generate a simulated network `net` with four non-network covariates `x1 - x4` which loads on two latent variables `lv1, lv2`.

```
set.seed(10)
nsamp = 50
lv1 <- rnorm(nsamp)
net <- ifelse(matrix(rnorm(nsamp^2) , nsamp, nsamp) > 1, 1, 0)
lv2 <- rnorm(nsamp)
nonnet <- data.frame(x1 = lv1*0.5 + rnorm(nsamp),
                     x2 = lv1*0.8 + rnorm(nsamp),
                     x3 = lv2*0.5 + rnorm(nsamp),
                     x4 = lv2*0.8 + rnorm(nsamp))
```

With the simulated data, we can define a `model` string with lavaan syntax that specifies the measurement model as well as the relationship between the network and the non-network variables. In this case, we are using `net` as a mediator between the two latent variables. Since data are generated randomly, the effects should be small overall.

```
model <- '
lv1 =~ x1 + x2
lv2 =~ x3 + x4
net ~ lv1
lv2 ~ net
'
```

Arguments passed to the `sem.net.edge.lsm` function includes the model, the dataset, and the latent dimensions. Note that `data` here should be a list with two elements, one being the named list of all

network variables and one being the dataframe containing non-network variables. A `summary` function can be used to look at the output.

```
data = list(network = list(net = net), nonnetwork = nonnet)
set.seed(100)
res <- sem.net.edge.lsm(model = model, data = data, latent.dim = 1)
summary(res)
path.networksem(res, 'lv2', c('net.dists'), 'lv1')
```

The output is shown below:

Model Fit InformationSEM Test statistics: 492.628 on 4 df with p-value: 0  
network 1 LSM BIC: 2244.546

=====

The SEM output:

lavaan 0.6.15 ended normally after 29 iterations

Estimator	ML
Optimization method	NLMINB
Number of model parameters	11
Number of observations	2500

Model Test User Model:

Test statistic	492.628
Degrees of freedom	4
P-value (Chi-square)	0.000

Model Test Baseline Model:

Test statistic	958.550
Degrees of freedom	10
P-value	0.000

User Model versus Baseline Model:

Comparative Fit Index (CFI)	0.485
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Tucker-Lewis Index (TLI)	-0.288
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Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-22209.465
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Loglikelihood unrestricted model (H1)	NA
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Akaike (AIC)	44440.930
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Bayesian (BIC)	44504.994
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Sample-size adjusted Bayesian (SABIC)	44470.045
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Root Mean Square Error of Approximation:

RMSEA	0.221
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90 Percent confidence interval - lower	0.205
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90 Percent confidence interval - upper	0.238
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P-value H_0: RMSEA <= 0.050	0.000
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P-value H_0: RMSEA >= 0.080	1.000
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Standardized Root Mean Square Residual:

SRMR	0.109
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Parameter Estimates:

Standard errors	Standard
Information	Expected
Information saturated (h1) model	Structured

Latent Variables:

	Estimate	Std.Err	z-value	P(> z )
lv2 =~				
x4	1.000			
x3	0.976	NA		
lv1 =~				
x2	1.000			
x1	0.642	NA		

Regressions:

Estimate	Std.Err	z-value	P(> z )
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```

net.dists ~
  lv1      -0.000    NA
lv2 ~
  net.dists  -0.000    NA

```

Variances:

	Estimate	Std.Err	z-value	P(> z )
.x4	2.856	NA		
.x3	1.501	NA		
.x2	1.722	NA		
.x1	2.490	NA		
.net.dists	0.553	NA		
.lv2	1.315	NA		
lv1	0.715	NA		

The LSM output:

```

=====
Summary of model fit
=====

```

```

Formula: network::network(data$network[[latent.network[i]]]) ~ euclidean(d = latent.dim)
<environment: 0x7fc473af4960>

```

Attribute: edges

Model: Bernoulli

MCMC sample of size 4000, draws are 10 iterations apart, after burnin of 10000 iterations.

Covariate coefficients posterior means:

	Estimate	2.5%	97.5%	2*min(Pr(>0),Pr(<0))	
(Intercept)	-0.67923	-0.83587	-0.5504		< 2.2e-16 ***

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Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

```

Overall BIC:      2244.546
Likelihood BIC:   2184.507
Latent space/clustering BIC:  60.03918

```

Covariate coefficients MKL:

	Estimate
(Intercept)	-1.117408

# Empirical Data Example

When embedding the LSM into the edge-based approach, one thing that needs to be considered is whether to model covariates predicting the social networks in the LSM framework or in the SEM framework. This is only a concern in the edge-based model since covariates need to be edge-based as well if using the LSM method, and it defies the purpose of simplicity if we consider the LSM in the actor-based approach. In this example, we will accommodate the covariates in the LSM framework within the edge-based approach. The dataset used in this example is the Florentine marriage dataset. The model is quite simple as shown below. Essentially, the observed marriage network is hypothesized to be based not only on the latent positions, but also on the non-network variable of wealth. Additionally, priorates is viewed as a predictor of the distance between latent positions of the marriage networks.

```
load("data/flomarriage.RData")

network <- list()
network$flo <- flomarriage.network
nonnetwork <- flomarriage.nonnetwork

model <- '
  flo ~ wealth
  priorates ~ flo + wealth
'
```

When fitting the model using the `sem.net.edge.lsm` function, the argument `type` and `latent.dim` are needed. Here, although the marriage network contains binary edges, the ordered argument is not needed since only the continuous latent distances will be used in the SEM.

```
data = list(network=network, nonnetwork=nonnetwork)
set.seed(100)
res <- sem.net.edge.lsm(model=model,data=data, type = "difference", latent.dim = 2, netstats.rescale = T,
data.rescale = T)
## results
summary(res)
```

In this model, the `latentnet` package is first used to estimate the LSM with the covariate of wealth. Then, the resulting latent positions of the marriage network, taking apart the effect of wealth, is hypothesized to be influenced by priorates and the effect is estimated through `lavaan`. Thus, the latent distances of the marriage network acts like a mediator between priorates and the observed network. The resulting estimates from both the SEM component and the LSM component are shown below.

Model Fit InformationSEM Test statistics: 0 on 0 df with p-value: NA  
network 1 LSM BIC: 259.7975

=====

=====

The SEM output:

lavaan 0.6.15 ended normally after 6 iterations

Estimator	ML
Optimization method	NLMINB
Number of model parameters	5
Number of observations	256

Model Test User Model:

Test statistic	0.000
Degrees of freedom	0

Model Test Baseline Model:

Test statistic	50.126
Degrees of freedom	3
P-value	0.000

User Model versus Baseline Model:

Comparative Fit Index (CFI)	1.000
Tucker-Lewis Index (TLI)	1.000

Loglikelihood and Information Criteria:

Loglikelihood user model (H0)	-700.431
Loglikelihood unrestricted model (H1)	-700.431
Akaike (AIC)	1410.863
Bayesian (BIC)	1428.589
Sample-size adjusted Bayesian (SABIC)	1412.737

Root Mean Square Error of Approximation:

RMSEA	0.000
90 Percent confidence interval - lower	0.000
90 Percent confidence interval - upper	0.000
P-value H_0: RMSEA <= 0.050	NA
P-value H_0: RMSEA >= 0.080	NA

Standardized Root Mean Square Residual:

SRMR	0.000
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Parameter Estimates:

Standard errors	Standard
Information	Expected
Information saturated (h1) model	Structured

Regressions:

	Estimate	Std.Err	z-value	P(> z )
priorates ~				
wealth	0.422	0.057	7.441	0.000
flo.dists ~				
wealth	0.000	0.063	0.000	1.000
priorates ~				
flo.dists	-0.000	0.057	-0.000	1.000

Variances:

	Estimate	Std.Err	z-value	P(> z )
.priorates	0.819	0.072	11.314	0.000
.flo.dists	0.996	0.088	11.314	0.000

The LSM output:

```
=====
Summary of model fit
=====
```

```
Formula: network::network(data$network[[latent.network[i]]]) ~ euclidean(d = latent.dim)
<environment: 0x7fc434ed5160>
Attribute: edges
```

Model: Bernoulli

MCMC sample of size 4000, draws are 10 iterations apart, after burnin of 10000 iterations.

Covariate coefficients posterior means:

Estimate 2.5% 97.5% 2\*min(Pr(>0),Pr(<0))

(Intercept) 5.0133 2.5627 7.9665 < 2.2e-16 \*\*\*

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Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Overall BIC: 259.7975

Likelihood BIC: 85.53086

Latent space/clustering BIC: 174.2666

Covariate coefficients MKL:

Estimate

(Intercept) 2.861026

To look at indirect effects, the following code can be used.

```
> path.networksem(res, "wealth", "flo.dists", "priorates")
predictor mediator outcome apath bpath indirect
1 wealth flo.dists priorates 2.976241e-21 -4.047181e-22 -1.204539e-42
indirect_se indirect_z
1 1.874237e-22 -6.42682e-21
```

The model is shown in this diagram below.

